التكامل المحدود

التكامل المحدود للدالة $f(x)$ من الحد a وحتى الحد b (بحيث $a < b$) هي:

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

أمثلة:

$$\int_{-4}^{4} (-4)\,dx = (-4x)\Big|_{-4}^{4} = (-4\cdot 4) - \big(-4\cdot(-4)\big) = -16 - 16 = -32$$

$$\int_0^1 (2x^3 - x)dx = \left(2\cdot\frac{x^4}{4} - \frac{x^2}{2}\right)\Big|_0^1 = \left(\frac{x^4}{2} - \frac{x^2}{2}\right)\Big|_0^1 = \left(\frac{1^4}{2} - \frac{1^2}{2}\right) - \left(\frac{0^4}{2} - \frac{0^2}{2}\right) = 0 - 0 = 0$$

$$\int_{-1}^{2} (-3x^2 + 2x + 1)dx = \left(-3\cdot\frac{x^3}{3} + 2\cdot\frac{x^2}{2} + x\right)\Big|_{-1}^{2} = (-x^3 + x^2 + x)\Big|_{-1}^{2}$$

$$= (-2^3 + 2^2 + 2) - (-(-1)^3 + (-1)^2 - 1) = -2 - 1 = -3$$

$$\int_{2.5}^{4} (2x - 5)^2\,dx = \int_{2.5}^{4} (4x^2 - 20x + 25)dx = \left(4\cdot\frac{x^3}{3} - 20\cdot\frac{x^2}{2} + 25x\right)\Big|_{2.5}^{4}$$

$$= \left(\frac{4}{3}x^3 - 10x^2 + 25x\right)\Big|_{2.5}^{4}$$

$$= \left(\frac{4}{3}\cdot 4^3 - 10\cdot 4^2 + 25\cdot 4\right) - \left(\frac{4}{3}\cdot 2.5^3 - 10\cdot 2.5^2 + 25\cdot 2.5\right)$$

$$= 25.333 - 20.833 = 4.5$$